Row Reduction Worksheet　　　　Math 202

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You should be able to do this whole packet in approximately 40 minutes, by working *efficiently* but without rushing.

(1) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -3 & -6 & 6 \\ -2 & 10 & 2 & -6 \end{array}\right]$

$(2 \;\; -6 \;\; 4 \mid 0) \leftarrow (-2)r_1$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -3 & -6 & 6 \\ 0 & 4 & 6 & -6 \end{array}\right] r_3^* = r_3 + (-2)r_1$$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & 1 & 2 & -2 \\ 0 & 4 & 6 & -6 \end{array}\right] \begin{array}{l} \\ -\tfrac{1}{3} r_2 \\ \\ \end{array}$$

$(0 \;\; -4 \;\; -8 \;\; 8) \leftarrow (-4)r_2$

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & -2 & 2 \end{array}\right] r_3^* = r_3 + (-4)r_2$$

consistent w/ pivot in ea. column
$\Rightarrow$ has <u>unique</u> sln.

$$\sim \left[\begin{array}{ccc|c} -1 & 3 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{array}\right] \begin{array}{l} r_1^* = r_1 - r_3 \\ r_2^* = r_2 + r_3 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 \end{array}\right] r_1 - 3r_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{cases} x_1 = 2 \\ x_2 = 0 \\ x_3 = -1 \end{cases}$$

the unique sln.

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(2) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ -4 & -4 & 6 & -6 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ -4 & -4 & 6 & -6 \end{array}\right]$$
$$(4 \quad 8 \quad -4 \quad 0) \leftarrow 2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ 0 & 4 & 2 & -6 \end{array}\right] r_3+2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

No pivot in col 3 $\Rightarrow \infty$-many slns

$$\sim \left[\begin{array}{ccc|c} 2 & 0 & -4 & 6 \\ 0 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right] r_1-r_2 \quad \Longleftrightarrow \quad \begin{cases} x_1 \quad -2x_3 = 3 \\ x_2 + 1/2\, x_3 = -\frac{3}{2} \\ x_3 \text{ free} \end{cases}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1/2 & -3/2 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} 1/2\, r_1 \\ 1/4\, r_2 \end{array} \quad \Longleftrightarrow \quad \begin{cases} x_1 = 3 + 2x_3 \\ x_2 = -\frac{3}{2} - \frac{1}{2}x_3 \\ x_3 \text{ free} \end{cases}$$

(3) Solve the system of equations with augmented matrix $\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ -4 & -4 & 6 & 0 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ -4 & -4 & 6 & 0 \end{array}\right]$$
$$(4 \quad 8 \quad -4 \mid 0)\ 2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 4 & 2 & 0 \end{array}\right] r_3+2r_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & -6 \end{array}\right] r_3-r_2$$

$\boxed{\text{No solution by theorem 2}}$

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(4) Give an example of the augmented matrix of a system of 3 equations in 3 variables ...
(*Hint:* you can make the systems as simple as you like.) rows coeff col's

(a) with a unique solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

(b) with no solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \end{array}\right]$$

(c) with infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

No pivot

(5) Give an example of the augmented matrix of a system of 2 equations in 3 variables ...
(*Hint:* you can make the systems as simple as you like.) rows coeff col's

(a) with no solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(b) with infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 2 \end{array}\right]$$

No pivot

(6) Use Theorem 2 to prove that you cannot write a system of 2 equations in 3 variables that has a unique solution. (*Hint:* your argument must consider all relevant reduced-echelon form matrices.)

$P \Leftrightarrow Q$ By theorem 2
unique solution $\Leftrightarrow$ has pivot in each coeff column

But

$\neg Q$
$$\left[\begin{array}{ccc|c} * & * & * & * \\ * & * & * & * \end{array}\right]$$

You cannot fit 3 pivots in two rows.

$\neg P$ Therefore the system cannot have a unique solution